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The Effectiveness of Geometric Representative Approach in Developing Algebraic Thinking of Fourth Grade Students

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Abstract

The main aim of this study was to examine the effectiveness of geometric representative approach in developing algebraic thinking of fourth grade students. Based on literatures in mathematics education, several procedures in developing algebraic thinking were identified. This study undertakes to (1) examine students' algebraic thinking and skills (2) investigate the ways of students in thinking algebraically (3)examine use of the geometric representation in developing algebraic thinking. During this study, the experimental group was taught the geometric representation approach whilst the control group was taught by conventional teaching approach. Findings indicated that the mean performance in algebraic thinking of the experimental group was higher than the control group. Findings suggested that the geometric representation approach enhances students' mathematical thinking.

***Keywords*:** Geometric representation; Algebric thinking

1. Introduction

The focus of the Korean curriculum is on the development of six thinking skills that lead to improved performance in algebra: (1) generalization, (2) abstraction, (3) analysis, (4) dynamism, (5) modeling, and (6) organization. The Chinese primary school curriculum has an overarching goal of understanding quantitative relationships with a focus on equations, equation solving, ratio and proportion, and variables and functions. In the national curriculum, the chief goal is the students' cognitive development through the enhancement of theoretical thinking skills (Beverly, 2004: 132:133).

Algebra is one strand of the elementary school mathematics. It focuses of algebraic activities, that help the students to make general statements about the behaviour of numbers, study the prosperities of computational operations , also make general geometric statements about shapes, transformations of shapes, geometric patterns; general statements about measures and frequencies of measures in statistical or other contexts. It is related to use working with a variety of algebraic materials or representations; and modelling and problem solving using a diversity of algebraic tools to represent the algebraic concept .(Irwin Kathryn & Britt Murray,2005: 170).

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Algebra is much more than just solving for x and y; instead, algebra is a way of thinking. Success in algebra depends on the ability to think in a variety of powerful ways that foster productive algebraic performance. When people think algebraically to solve problems, various habits of thinking come into play, such as doing-undoing, building rules to represent functions, and abstracting from computation. Curricula, specifically instructions can serve to demystify algebra by providing activities that foster these sorts of thinking in students (Cai Jinfa, et.al, 2005:2).

What is algebraic thinking? Although it may include variables and expressions, algebraic thinking has a broader and different connotation than the term algebra. The term algebraic thinking can be defined as "the use of any of a variety of representations that handle quantitative situations in a relational way. Another definition of algebraic thinking was "the ability to operate on an unknown quantity as if the quantity was known, in contrast to arithmetic reasoning which involves operations on known quantities. The algebraic thinking could be considered to be the "capacity to represent quantitative situations so that relations among variables become apparent". These definitions are all similar, and we use them to guide our own use of the term "algebraic thinking" (Steele & Johanning, 2004: 65).

Algebraic thinking consists of more than just learning how to solve for the variables x and y; it helps students think about mathematics at an abstract level, and provides them with a way to reason about real-life problems. You can explore three components of algebraic thinking: (1) making generalizations, (2) conceptions about the equals sign, and (3) reasoning about unknown quantities.

Algebraic thinking is defined and described as useful ways of thinking about mathematical content. In this research we focused on selected features of algebraic thinking, for examples, describing a rule of patterns, describing change in a process or a relationship, and representing/ solving word problems.

Expecting students in elementary grades to think algebraically is not an issue among researchers these days. Based on recent research on learning, there are many obvious and widely accepted reasons for maintaining this expectation (Cai et al. 2005, 10). Nonetheless, we raise the 3 question in order to offer a less obvious reason for developing algebraic ideas in the earlier grades, namely that resistance to algebra in middle and high school would be reduced if we could remove the misconception that arithmetic and algebra are disjoint subjects. Traditionally, most school mathematics curricula separate the study of arithmetic and algebra—arithmetic being the primary focus of elementary school mathematics and algebra the primary focus of middle and high school mathematics. There is a growing consensus, however, that this separation makes it more difficult for students to learn algebra in the later grade (Cai & John, 2008: 2-3.)

The mathematics teachers should provide various instructional efforts for students to learn algebra more easily at the junior high school when students were introduced to learning of algebra. Many concrete operational activities should be given to reduce a cognitive gap between algebra and arithmetic. Also, in elementary school, many prerequisite activities should be provided to establish a good foundation for learning algebra (Hee, 2004:88).

On the other side, teaching algebra and solving word problems in algebra depends on the mathematics representation. It means that the students should be able to use the mathematical tools to represent the patterns or represent the word problem algebraically (Kristen & Rebacca, 2000: 124). One of the most interesting aspects of the elementary school mathematics texts and workbooks used in Singapore is representing the algebraic concepts and word problem using tables or diagrams. It helps the students to make a plan to solve and build correct image about the algebraic concepts, also it helps the students to make the generalization that it is considered the main of aim in teaching algebra (Beckmann,2004:42).

Geometric representation plays extremely important roles in mathematics in general and in teaching algebra as strand of mathematics education. These important roles can be categorized as follows: (1) Thinking through what is represented in mathematics content which helps the students to build the different strategies of algebraic thinking, (2) Recording what was thought through representations (as a method of recording algebraic concepts), and (3) An important method for mathematical communication which is also the geometric representation that helps the students to build the correct images about algebraic concepts (Radford, 2010:4).

This study was conducted to achieve the following objectives;

1. to describe students algebraic thinking skills,
2. to describe the geometric representation approach in teaching algebra,
3. to assess students' algebraic thinking skills, and
4. to investigate the effectiveness of the geometric representation approach in developing algebraic thinking skills.

The main research questions in this study are:

1. How to use the geometric representation approach in teaching algebra? and
2. What is the effectiveness of the geometric representation approach in developing algebraic thinking skills of fourth grade students?

Significance of study:

1. Helping curriculum designers to develop teaching algebra in primary school,
2. Helping teachers to develop students' algebraic thinking through selecting activities and using the geometric representation in teaching algebra, and
3. Helping teachers to select appropriate evaluation techniques for assessing students' mastery of algebraic thinking skills.

2. Methodology

This study adopted the experimental research design, where the experimental group was taught the geometric representation approach and the control group was taught according the traditional approach. The algebraic thinking skills test was administered to the two groups, one before and another after the experimental treatment in order to compare their algebraic thinking skills and investigate the effects of the suggested approach.

In preparing the Algebraic thinking skills test**,** many skills in algebra thinking were identified (Cai, et.al, 2005:10, Cai& John, 2008: 2-3). It includes the following: (1) Understanding the patterns, (2) Use algebra symbol, (3) Use math- geometry model, (4) Analyze the change, the relations and the equations, and (5) Solve word problems algebraically and geometrically. The test given after the intervention was the multiple-choice items test. There were 20 items, 4 items assessing each sub-skill. A pilot study was conducted on fourth grade students (57 students). Following that, two weeks later, the test was administered again to the same group of students. The reliability was calculated using the correlation was 0.862, hence the reliability of the instrument is at acceptable value.

Also to achieve the main aim of this study, the model of the geometric representation approach was designed. It depends on three steps as follows: (1) Opening the idea - it is related to present the problem and read it, (2) Developing the idea - it is related to representing the problem using the figures, tables, and others. The students solve the problem using various strategies, (3) Closing the idea - it is related to rethink in the representation and solve again. Also present the summary about what they learn from the problem.

Two groups of fourth grade students were sampled out: experimental group = 34 students, controlling group = 31 students. The pre test was applied. There were no differences between the two group mean achievements. After that, the experimental group was taught through geometric representation approach, with the controlling group was taught through traditional strategy. The strand of algebra in fourth grade include the following subjects(1)Represent the numerical statements,(2) Write in algebraic form,(3) Make a plan for solving word problem, (4) Discover the rule from the table, and (5) The equation and function.

The experiment of this study took place in the second term in 2009/2010 of the school year. Through our follow the school and teacher we observed the students in class. They used the math tool to represent the concept, and use the tools to represent their idea. Finally the post test was administered again. The data was analysed using the Statistical Package for Social Science (SPSS).

3. Findings

The findings of this study are related to the following hypothesis: There are statistically significant differences between the means of the two groups scores in the algebraic thinking skills test (in general and for each sub-skill). The means and standard deviation were calculated as in Table 1. The following figure depicts comparisons of the algebraic test performance between the two groups.

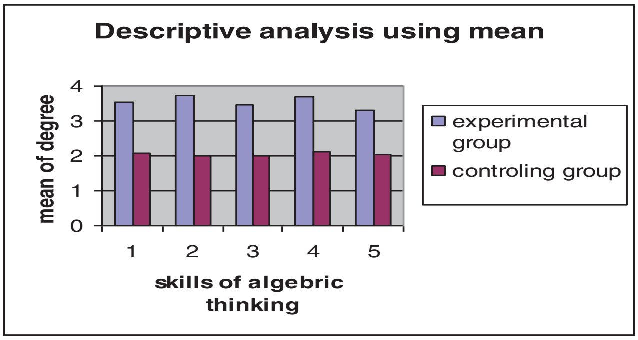


Figure 1: Descriptions of Algebraic Thinking Skills of the experimental and control groups

Findings as indicated in table 1 showed that there are differences among the means of overall and sub-scales in algebraic thinking skills between the experimental and control groups. The mean of the experimental group is more than the mean of the control group. Specifically, thinking skills for use of algebra symbol (sub-scale 2) showed highest mean (3.725) in favour of the experimental group. In addition experimental group of students scored higher in other subscale such as “change and the relation”, “understanding patterns” and “use of math-geometry model. In the comparison analyses, the independent t-test was used to establish the statistically significant differences between the means of the two groups as following:

Table 1 Results of t-test

	Thinking Skills	Groups	N	M	SD	t-test	df	Significant Value	Effect size
1	Understanding the patterns	experimental	34	3.542	0.982	4.796	63	0.01	1.19
		control	31	2.089	1.421				
2	Use algebra symbol	experimental	34	3.725	1.005	5.733	63	0.01	1.42
		control	31	1.982	1.412				
3	Use math- geometry model	experimental	34	3.460	0.823	5.626	63	0.01	1.40
		control	31	1.999	1.231				

4	Change and the relations	experimental	34	3.683	1.100	5.009	63	0.01	1.24
		control	31	2.106	1.421				
5	Word problems	experimental	34	3.300	0.898	4.734	63	0.01	1.18
		control	31	2.040	1.223				
	Total	experimental	34	17.710	2.531	9.204	63	0.01	2.29
		control	31	10.210	3.895				

From Table 1, there were significant differences in the mean performance of each of the algebraic thinking sub-scale and the total algebraic performance statistically ($\alpha \leq 0.01$). Findings were in favour of the experimental group. These results suggest that in order to develop algebraic thinking, teacher should use many mathematical representations during teaching of mathematics. This helps the students to understand algebraic problems and the problem-solving process as well. Hence mathematical representation enhances students' problem- solving abilities (Hee, 2004:91-92). Also the teachers of experimental group used a lot of various approaches in representing the algebra concepts, skills and facts.

4. Conclusion

Using the geometric representation helps the students in the experimental group to read the algebraic word problem, represent it using the tools through table, diagrams and graphs. It helps the students to understand the algebraic concepts through building correct image and represent it by the mathematical tools. Also it helps the students to think algebraically through the use of correct symbols, build the geometric models for the concepts, deduce the roles, discover the relation in the presented patterns, build the new patterns, and solve word problems. These findings are consistent with Radford (2010) which also concluded that mathematical representation helped the students to develop the algebraic thinking skills required (Radford, 2010, 17). Thus using the geometric representations help students in presenting the concepts in action.

References

Beverly J. Ferrucci (2004). Gateways to Algebra at the Primary Level, the Mathematics Educator, Vol.8, No.1, pp (131 – 138).

Cai, Jinfa, Hee Chan Lew, Anne Morris, John C. Moyer, Swee Fong Ng, and Jean Schmittau. (2005) “The Development of Students' Algebraic Thinking in Earlier Grades: A Cross-Cultural Comparative Perspective.” Zentralblatt fuer Didaktik der 37 pp. 5 – 15.

Cai Jinfa & John Moyer (2008). Developing Algebraic Thinking in Earlier Grades: Some Insights from International Comparative Studies, National Council of Teachers of Mathematics, Yearbook.

Hee-Chan Lew (2004). Developing Algebraic Thinking in Early Grades: Case Study of Korean Elementary School Mathematics, the Mathematics Educator, Vol.8, No.1, 88 – 106.

Herbert Kristen & Rebacca Brown (2000). Patterns as Tools for Algebraic Reasoning, National council of teachers of mathematics, Reston, VA: Author, pp 123-128.

Irwin Kathryn & Britt Murray (2005). The Algebraic Nature of Students’ Numerical Manipulation in the New Zealand Numeracy Project, Educational Studies in Mathematics, Vol.58, pp 169–188.

National Council of Teachers of Mathematics (2000). Principles and Standards for School Mathematics. Reston, VA: Author

Radford Luis (2000). Signs and Meaning in Students’ Emergent Algebraic Thinking: A Semiotic Analysis, Educational Studies in Mathematics, Vol.42, pp. 237–268.

Radford Luis (2010). Algebraic Thinking from a Cultural Semiotic Perspective, Research in Mathematics Education, Vol. 12, No. 1, 1_19

Steele Diana & Johanning Debra (2004) Schematic –Theoretic View of Problem Solving and Development of Algebraic Thinking, Educational Studies in Mathematics, Vol.57, pp. 65–90.

Sybilla Beckmann (2004). Solving Algebra and Other Story Problems with Simple Diagrams: a Method Demonstrated in Grade 4–6 Texts Used in Singapore, The Mathematics Educator, Vol.14 (1), pp. 42–46.

Algebraic thinking skills test

Algebraic thinking skills test – fourth grade students

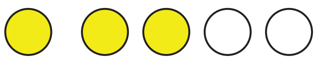
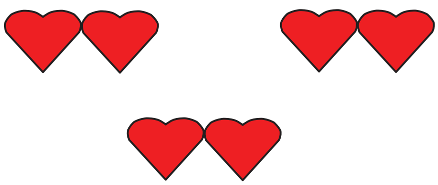
Name:……………………………………………………….…………………
School:………………………………………………………………………
Date: …………………………………………………………………..

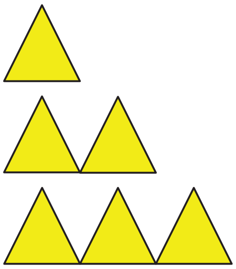
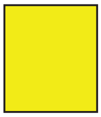
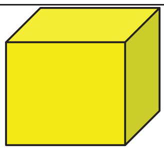
Test's instruction

- The test includes 20 items.
- You can use the remark part to make a plan or solve the questions
- There is one answer that is correct.
- Use the following answer sheet and shadd in the forent of the correct answer

No.	A	B	C	D	No.	A	B	C	D
1					11				
2					12				
3					13				
4					14				
5					15				
6					16				
7					17				
8					18				
9					19				
10					20				

Sample Questions - Algebraic thinking skills test

No.	items	Multipulchoses	Remark
1	The missing number in the following pattern is: 1, 1 , 2 , 3, 5,......., 13	(a) 7 (b) 8 (c) 10 (d) 12	
2	The shadod part is closed with...... 	(a) 30% (b) 40 % (c) 60% (d) 100%	
3	Observe the following drawing  This shape is represented to	(a) $3 \times 2 = 6$ (b) $2 \times 3 = 6$ (c) $1 \times 6 = 6$ (d) Others	

<table>
<tr><th>No.</th><th>items</th><th>Multipulchoses</th><th>Remark</th></tr>
<tr><td>4</td><td>Read the following and deduce the (x)...
<table><tr><td>6</td><td>5</td><td>11</td><td>x</td><td>50</td></tr><tr><td>18</td><td>15</td><td>33</td><td>30</td><td>150</td></tr></table></td><td>(a) 1
(b) 10
(c) 60
(d) 90</td><td></td></tr>
<tr><td>5</td><td>
İf each traiangle has 3 side, and you construct (x) times of traiangles, so the numbers of sides will be recoognized by the formula:</td><td>(a) 3 +x
(b) 3 + 3x
(c) 3x
(d) x</td><td></td></tr>
<tr><td>6</td><td>
As you see the square its length = y , complete the missing
<table><tr><td>Lenght</td><td>y</td><td>2</td><td>3</td><td>.............</td></tr><tr><td>The rea</td><td>y^2</td><td>4</td><td>9</td><td>64</td></tr></table></td><td>(a) 4
(b) 8
(c) 16
(d) 4096</td><td></td></tr>
<tr><td>7</td><td>• The length of Ahmed = 132 cm
• The length of Mohammed = 137 cm
• The lenght of Ibraheem = ... cm
İf the length is more less than The lenght of Mohammed, and more than The lenght of Ahmed, so the missing lenght is</td><td>(a) 131
(b) 137
(c) 138
(d) Otherwise</td><td></td></tr>
<tr><td>8</td><td>Rasheid and Mansour shared (R.S) 410 between them. Rasheid received (R.S) 100 more than Mansour. How much money did Mansour receive?</td><td>(a) 100
(b) 155
(c) 205
(d) 305</td><td></td></tr>
<tr><td>9</td><td>Sammy had (R.S) 47. After paying for 3 kg of Apple, he had (R.S) 20 left. Find the cost of 1 kg of prawns.</td><td>(a) 9
(b) 27
(c) 67
(d) 81</td><td></td></tr>
<tr><td>10</td><td>If 23 + $\boxed{}$ = 30
So $\boxed{}$ = ...</td><td>(a) 6
(b) 8
(c) 23
(d) 53</td><td></td></tr>
<tr><td>11</td><td>
Cube</td><td>(a) 27
(b) 28
(c) 64
(d) 125</td><td></td></tr>
</table>

<table>
<tr><th>No.</th><th>items</th><th>Multipulchoses</th><th>Remark</th></tr>
<tr><td></td><td>Observe how to calculate the volume of the cube, then deduce the missing number?
<table>
<tr><td>Lenght of its edge</td><td>(a) cm</td><td>1</td><td>2</td><td>3</td><td>More than 3, and less than 4</td></tr>
<tr><td>İts volume</td><td>(a^3) cm^3</td><td>1</td><td>8</td><td>27</td><td>.............</td></tr>
</table></td><td></td><td></td></tr>
<tr><td>13</td><td>Samier had 40 (r.s). he is paying gift for his brohrt. The rimnder was 12 (r.s)?
The correct represent in comutational operation is:</td><td>(a) 40 – 12 = ...
(b) 12 + ... = 40
(c) 40 – ......= 12
(d) Otherwise</td><td></td></tr>
<tr><td>14</td><td>Observe and deduce the missing number in the following pattern:

1 , 8 , 27 , ... , 125</td><td>(a) 32
(b) 36
(c) 64
(d) 100</td><td></td></tr>
</table>